K-6 Teacher Beliefs About the Learning and Teaching of Mathematics

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This paper has been developed as part of a continuing investigation of the use of manipulatives in mathematics learning and teaching by K - 6 teachers. The focus is the beliefs held by teachers about mathematics, mathematics learning and mathematics teaching. The grouping of particular beliefs which allows the development of a profile of these teachers is considered.

Introduction

In the past decade, there has been considerable comment related to the importance of teacher beliefs in the mathematics classroom (Barnett & Sather, 1992; McLeod, 1992; Pajares, 1992; Weissglass, 1992). This investigation of the beliefs held by 252 teachers, working during 1995 in government primary schools in the south western suburbs of Sydney, adds to this growing literature and provides some basis from which decisions concerning mathematics curriculum and pedagogy might be made. In addition, it provides a basis for future study of the actual practices of such teachers.

Teacher beliefs and mathematics

There appears to be more than the knowledge of facts and formulas involved in one's ability to do mathematics (Garofalo, 1989). In recognising this, the complex interaction of the affective and the cognitive domains in the learning and teaching of mathematics continues as an important area of investigation for mathematics educators, both nationally and internationally (Howard, 1994; McLeod, 1992; Southwell, 1995; Southwell & Khamis, 1992). In such investigations a belief can be defined as "*any simple proposition, conscious or unconscious, inferred from what a person says or does, capable of being preceded by the phrase: 'I believe that...*'" (Rokebach, 1968, p.2). Pajares (1992, p.327) suggests that beliefs are "*the single most important construct in educational research*".

All teachers have beliefs about mathematics and its learning and teaching. Baroody (1987, p.5) notes that

> *... everyone has a set of beliefs about how mathematics is learned. These beliefs have an influence on all aspects of teaching. They govern what is considered appropriate to include in a curriculum and when topics should be taught; they determine the importance an educator attributes to gauging readiness skills or exploiting children's curiosity and interests; and they effect how educators teach skills and concepts, evaluate progress and remedy difficulties. In brief, whether conscious or not, beliefs about mathematics learning guide decision making and, in the end, influence our effectiveness as mathematics educators.*

Beliefs about mathematics, and one's ability to do mathematics, influence how people study and perform mathematics (Garofalo, 1989). Personal beliefs incorporate one's world view of mathematics and influence the context in which mathematics is done (Schoenfeld, 1985). One way of examining teachers' espoused beliefs about mathematics has been to categorize them into those related to the nature of mathematics, the learning of mathematics and the teaching of mathematics.

One study which investigated the mathematical beliefs of students and teachers (Southwell & Khamis, 1992) reported that teachers firstly need to identify their held beliefs about the nature of mathematics and the learning and teaching of mathematics, and, secondly, to consider the influence of these beliefs on the students' learning environment (Barnett & Sather, 1992). The present investigation of teacher beliefs links them to the teachers' major task of facilitating the mathematics education of their students.

Methodology

The data upon which this paper is based were collected using a specifically designed questionnaire consisting of both multiple choice and open-ended questions covering the following areas:

1. respondent demographics such as gender, age, position in school, nature of initial teacher education, length of teaching experience, class(es) currently taught, class size, classes taught over the last ten years;
2. use of concrete materials in mathematics learning and teaching such as which materials are used, why, how often and how they are used, and the areas of mathematics in which they are used;
3. beliefs about mathematics, mathematics learning and mathematics teaching.

Section 3 of the questionnaire sought responses to a set of teacher belief statements which were derived from Australian Education Council (1991), Barnett & Sather (1992), Mumme & Weissglass (1991), Weissglass (1992) and Wood, Cobb & Yackel (1992). During its preparation, the questionnaire went through a number of iterations as a result of input from several mathematics educators.

In August, 1995, the questionnaire was posted, with reply paid envelopes, to 25 government primary schools in the Metropolitan South West Region of the NSW Department of School Education. The schools were randomly chosen from the 150 government primary schools in the Region. Sufficient numbers of the questionnaire were posted to cover all teachers at these schools. Two hundred and fifty-two completed questionnaires were received.

Results

Demographic data

Of the 252 respondents, 218 (87%) were female. The (former) Metropolitan South West Region of the New South Wales Department of School Education was often referred to as the 'nursery' for the State in terms of the relative inexperience of its teachers. Hence, it is somewhat surprising to note that 49 (19%) of the respondents had in excess of 20 years teaching experience, while 66 (26%), 62 (25%), 67 (27%) had from 11 to 20 years, 6 to 10 years, 1 to 5 years teaching experience respectively. Only 8 respondents (3%) had less than one year of teaching experience. The school positions held by the respondents are reported in Table 1.

Table 1 School positions held n=252

Current position	Number	Percentage
Principal	11	4
Deputy Principal	10	4
Assistant Principal	19	8
Executive Teacher / AST	23	9
Classroom Teacher	155	62
ESL Teacher	31	12
Other Teacher	3	1

Fifteen (6%) of the respondents described themselves as two year trained teachers, 78 (31%) as three year trained with a Diploma in Teaching, 14 (6%) as three year trained with a Bachelor of Teaching, 110 (44%) as four year trained with a Bachelor of Education and 11 (4%) as teachers with more than four years of training. Another 24 (9%) respondents described their teacher education status in other ways such as Diploma / Graduate Diploma.

Respondents were asked to indicate the class or classes in which they were currently teaching mathematics. These responses were grouped into the four categories K - 2, 3 - 4, 5 - 6 and Nil Response. There were 33 (13%) Nil Responses to this question, perhaps indicating that these respondents were not teaching mathematics during 1995. Details are given in Table 2.

Table 2 Mathematics teaching during 1995 n=252

Range of Years	Number	Percentage
Kindergarten to Year 2	98	39
Years 3 and 4	61	24
Years 5 and 6	60	24
Nil response	33	13

Statements of beliefs - Frequency of teacher responses

Table 3 reports the overall frequency of teacher responses to the 20 statements of beliefs concerning the nature of mathematics and mathematics learning and teaching. Teachers were asked to respond to each belief statement by indicating whether they disagreed with the statement, were undecided about it, or agreed.

Table 3
Statements of belief - Frequency of responses (Percentage of respondents) n=252

Statement of belief	Disagree	Undecided	Agree
Mathematics			
1. Mathematics is computation	87 (36)	41 (17)	116 (48)
2. Mathematics problems given to children should be quickly solvable in a few steps	138 (57)	45 (18)	61 (25)
3. Mathematics is the dynamic searching for order and pattern in the learner's environment	11 (4)	62 (25)	178 (71)
4. Mathematics is no more sequential a subject than any other	142 (57)	40 (16)	66 (27)
5. Mathematics is a beautiful, creative and useful human endeavour that is both a way of knowing and a way of thinking	24 (10)	75 (31)	147 (60)
6. Right answers are much more important in mathematics than the ways in which you get them	214 (86)	20 (8)	14 (6)
Mathematics learning			
7. Mathematics knowledge is the result of the learner interpreting and organising the information gained from experiences	5 (2)	12 (5)	234 (93)
8. Children are rational decision makers capable of determining for themselves what is right and wrong	90 (36)	76 (30)	85 (34)
9. Mathematics learning is being able to get the right answers quickly	212 (85)	19 (8)	20 (8)
10. Periods of uncertainty, conflict, confusion, surprise are a significant part of the mathematics learning process	20 (8)	26 (10)	206 (82)
11. Young children are capable of much higher levels of mathematical thought than has been suggested traditionally	17 (7)	83 (33)	151 (60)
12. Being able to memorise facts is critical in mathematics learning	71 (29)	57 (23)	120 (48)
13. Mathematics learning is enhanced by activities which build upon and respect students' experiences	1 (0)	7 (3)	244 (97)
14. Mathematics learning is enhanced by challenge within a supportive environment	1 (0)	6 (2)	244 (97)
Mathematics teaching			
15. Teachers should provide instructional activities which result in problematic situations for learners	9 (4)	23 (9)	215 (87)
16. Teachers or the textbook - not the student - are the authorities for what is right or wrong	176 (71)	56 (23)	16 (6)
17. The role of the mathematics teacher is to transmit mathematical knowledge and to verify that learners have received this knowledge	94 (38)	44 (18)	109 (44)
18. Teachers should recognise that what seem like errors and confusions from an adult point of view are children's expressions of their current understanding	8 (3)	36 (14)	206 (82)
19. Teachers should negotiate social norms with the children in order to develop a cooperative learning environment in which children can construct their knowledge	8 (3)	56 (23)	184 (74)
20. It is unnecessary, even damaging, for teachers to tell students if their answers are correct or incorrect	207 (83)	32 (13)	12 (5)

As with questionnaire research in general, a limitation to this study is that all results must be considered in the context that responses to the belief statements are dependent on the interpretations given to them by the respondents.

Beliefs about mathematics

Forty-eight percent of the respondents indicated a belief that *mathematics is computation*. In spite of this, 86% disagreed with *right answers are much more important in mathematics than the ways in which you get them*. As well, 71% believe that *mathematics is the dynamic searching for order and pattern in the learner's environment* and 60% believe that *mathematics is a beautiful, creative and useful human endeavour*. Interestingly, 57% of respondents disagree with the statement that *mathematics is no more sequential a subject than any other*.

Beliefs about mathematics learning

Ninety three per cent of respondents believe that *mathematics knowledge is the result of the learner interpreting and organising the information gained from experiences*. This is supported by 97% of respondents who believe that *mathematics learning is enhanced by activities which build upon and respect children's experiences* and *by challenge within a supportive environment*. Further, 82% of the respondents saw that *periods of uncertainty, conflict, confusion, surprise are a significant part of the mathematics learning process*.

Almost half (48%) of the respondents agree that *being able to memorise facts is critical in mathematics learning*. Sixty percent agreed that *young children are capable of much higher levels of mathematical thought than has been suggested traditionally* while 85% disagreed that *mathematics learning is being able to get the right answers quickly*. This can be contrasted with the 57% of respondents who disagreed that *mathematics problems given to children should be quickly solvable in a few steps*.

Of particular interest is the belief that *children are rational decision makers capable of determining for themselves what is right and wrong* where 36% of respondents disagreed and 34% agreed. This item will be the subject of further analysis.

Beliefs about mathematics teaching

The belief that *teachers should provide instructional activities which result in problematic situations for learners* was agreed to by 87% of the respondents with 71% disagreeing that *teachers or textbooks - not the student - are the authorities for what is right or wrong*. The majority (74%) of respondents agreed that *teachers should negotiate social norms with the children in order to develop a cooperative learning environment in which children can construct their knowledge*. However, this finding must be tempered by consideration about *whose* social norms are to be negotiated.

The belief that *the role of the mathematics teacher is to transmit mathematical knowledge and to verify that learners have received this knowledge* polarised the respondents with 44% agreeing with the statement and 38% disagreeing. On the other hand, 83% of the respondents disagreed with the statement that *it is unnecessary, even damaging, for teachers to tell students if their answers are correct or incorrect*. This would seem to contrast with the response to the statement concerning authorities for what is right and wrong.

Eighty-two percent of respondents agree that *teachers should recognise that what seem like errors and confusions from an adult point of view are children's expressions of their current understanding*. This would appear to be quite a shift from a traditional view of errors as **wrong** answers which indicate children's ability levels.

The interconnectedness of the belief statements

Through a chi-square analysis of the responses to these belief statements, some statistically significant relationships ($p<0.05$) between the statements have been identified.

Statistically significant results

The belief that *mathematics is computation* (1) is significantly related to three beliefs:

- *mathematics problems given to children should be quickly solvable in a few steps* (2) ($p<0.001$)
- *being able to memorise facts is critical in mathematics learning* (12) ($p<0.0005$)
- *the role of the mathematics teacher is to transmit mathematical knowledge and to verify that learners have received this knowledge* (17) ($p<0.00005$).

That these four beliefs are related is not surprising in that all four represent facets of what might be called a transmission or absorption approach to mathematics learning and teaching (Baroody, 1987; Wright, 1991). Relationships between pairs of these four belief statements are also statistically significant: 2 and 12 are related ($p<0.02$); 2 and 17 ($p<0.002$) and 12 and 17 ($p<0.0001$).

Another group of belief statements which display statistically significant relationships are *children are rational decision-makers capable of determining for themselves what is right and wrong* (8) with:

- *mathematics is no more sequential a subject than any other* (4) ($p<0.05$)
- *being able to memorise facts is critical in mathematics learning* (12) ($p<0.02$).

However, 4 and 12 are not related significantly to each other.

Profiling teachers through espoused beliefs

The above statistical analysis suggests that the set of beliefs: 1, 2, 12 and 17 when held by teachers, tend to be held strongly as a group. These beliefs can be characterised as *transmission* beliefs. On the other hand, there is a set of beliefs which are held by the majority of the respondents but which are not held as strongly, as a group of beliefs, as the transmission set. Nonetheless, it is possible to pair these beliefs if 60% of the respondents (an arbitrary limit chosen by the researchers) agree with both belief statements, disagree with both or agree with one and disagree with the other. This enables a profile of the group of teachers to be inferred.

For example, 86% of all respondents disagreed with the belief statement that *right answers are much more important in mathematics than the ways in which you get them* (6). While this result itself is spectacular enough, of further interest is the way in which this result is linked to the response rates to other belief statements. Of all the respondents, the percentage which disagree with statement 6 and:

- agree with *mathematics is the dynamic searching for order and pattern in the learner's environment* (3) is 63%;
- agree with *mathematics knowledge is the result of the learner interpreting and organising the information gained from experiences* (7) is 82%;
- disagree with *mathematics learning is being able to get the right answers quickly* (9) is 77%;
- agree with *periods of uncertainty, conflict, confusion, surprise are a significant part of the mathematics learning process* (10) is 72%;
- agree with *mathematics learning is enhanced by activities which build upon and respect students' experiences* (13) is 84%;

- agree with *mathematics learning is enhanced by challenge within a supportive environment* (14) is 84%;
- agree with *teachers should provide instructional activities which result in problematic situations for learners* (15) is 75%;
- disagree with *teachers or the textbook - not the student - are the authorities for what is right or wrong* (16) is 66%;
- agree with *teachers should recognise that what seem like errors and confusions from an adult point of view are children's expressions of their current understandings* (18) is 73%;
- agree with *teachers should negotiate social norms with the children in order to develop a cooperative learning environment in which children can construct their knowledge* (19) is 66%;
- disagree with *it is unnecessary, even damaging, for teachers to tell students if their answers are correct or incorrect* (20) is 70%.

All of these beliefs, perhaps with the exception of 20, display a child-centred approach to teaching and learning mathematics. That so many of the respondents adhere to these beliefs both individually and as a group is heartening to mathematics educators who have worked for a shift towards a child-centred approach over the last 20 years.

This analysis indicates that grouping belief statements can help lead to the development of profiles for the respondents. It is anticipated that further analysis of the data, including the investigation of the relationship between these espoused beliefs and the classroom practice of the teachers, will add to our knowledge in this area.

Conclusion

This paper has investigated the espoused beliefs of teachers and pointed to the need for further investigation. A natural extension of this investigation is to include the beliefs of secondary mathematics teachers in the broader research agenda announced by Perry & Howard (1994).

The results of this study suggest that there is a need for mathematics educators to identify more clearly the place of memorising facts in the learning of mathematics, to continue discussion about the role of computation in mathematics and to seek more balance between students obtaining the right answers in mathematics and the importance of the mathematical processes that they go through.

It is not only the mathematics learning and teaching which needs to be investigated further. It is also necessary for mathematics educators to consider the complex web of social interactions which occur in the mathematics classroom within the framework of negotiated social norms and to identify the constraints and supports within this social context which determine the degree to which the teacher is able to enact his / her beliefs about the learning and teaching of mathematics.

Given the overwhelming support for mathematics learning which is built upon students' experiences and takes place in a challenging and supportive environment characterised by periods of uncertainty, conflict, confusion and surprise, it is pertinent to ask how one establishes and maintains such an environment. Many of these characteristics have been associated with what have become known as *constructivist classrooms* (Lo, Wheatley & Smith, 1994; Simons, 1995; Wood, Cobb, Yackel & Dillon, 1993). A further follow up to this study would be to undertake observations in the mathematics classes of K - 6 teachers to ascertain the degree to which their espoused beliefs are evident in practice.

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